

LOGIC 2

FORMAL LOGIC

WORKBOOK



Tracking Down Truth with
FORMAL LOGIC

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CINDY M. FELSO

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INTRODUCTION

Hello and welcome to *Logic 2* by Veritas Press! *Logic 2* is a course in *formal logic*, which you'll learn about in the first few chapters. *Logic 2* focuses on concepts and skills that can equip you to become a better thinker. Learning to think better is valuable because it helps direct us toward, and keep us on, the path to truth.

Logic 2 is about more than learning to think better. The book also focuses on developing strong character. Strong character, doubtless you already know, develops neither by accident nor overnight. Better thinking is valuable in itself, of course. It's best, though, when paired with loving attitudes and behaviors.

The book in your hands is the *Logic 2* student *workbook*, the companion to the student *textbook*. In the workbook, you'll practice applying what's covered in the textbook. The workbook will also call you to explore how the book's content connects to the "real world" outside. Some of the questions in the workbook can be answered after a simple reading of a chapter. Some of the questions, however, will require deep thought and honest reflection.

It's on that point that the authors of this workbook would like to make a deal with you. We've worked hard to ask thoughtful, relevant questions. We ask *you* to work hard to provide thoughtful, reflective answers. Sure, you'll be tempted now and then to write only what's needed to get by. (We *all* do that from time to time!) We ask you to resist the temptation. The textbook has a lot of useful content. It's the workbook, though, that'll help impress its lessons into your head, heart, and hands.

For each chapter in the textbook, you'll find a corresponding set of exercises in this workbook. Each set of exercises is divided into five sessions, 1-5. Every time you come to a session 1, you can expect a certain sort of questions. The same is true about sessions 2-5. (All session 1 material looks a lot alike, that is, as does all session 2 material, etc. You get the idea.) In the next section below, you'll see what kinds of questions appear in each session. You'll also find descriptions of the question types you'll encounter.

Before starting a set of exercises, be sure to read its chapter, top to bottom. Pause here and there to check your understanding. Natural places to pause are the section breaks, where the topic or focus changes. After each section, consider asking yourself a couple of questions. "What did I just read? Can I summarize it in my own words? Anything in the section confusing or otherwise unclear?"

If the answer to the last question is yes, then reread the section. Ask a teacher or parent for assistance if that doesn't solve the problem. What you don't want to do is to leave a chapter, confused about its content. Chapters tend to build on one another, so you don't want to start building on sinking sand!

So, what kinds of questions will you encounter in each set of exercises? Let's take a look.

What's in each session?

1. In every session 1, you'll find questions about Terms & Concepts and Big Ideas. You'll give straightforward definitions of new and reviewed terms. You'll also respond to short-answer questions about the chapter's main ideas.
2. In every session 2, you'll find Discussion or Demonstration questions. You'll answer questions that deepen, extend, or apply chapter content.
3. In every session 3, you'll find Discussion or Demonstration questions. You'll answer questions that deepen, extend, or apply chapter content.
4. In every session 4, you'll find Discussion or Demonstration questions and a Quiz. You'll answer questions that deepen, extend, or apply chapter content. You'll also answer matching, true–false, or short-answer questions about book content.
5. In every session 5, you'll find Puzzles & Perspectives. You'll respond to questions that explore a chapter topic, or a related topic, in greater depth.

A last word before you begin

In many chapters of the student textbook, you'll notice one or more special vocabulary words. Most often, those words are indicated by a **blue** typeface. The textbook has no glossary for those vocabulary words, but they're all defined in context. Where the authors introduce a special term, that is, they define it nearby.

Still, you may find it helpful to create your own glossary of special vocabulary. You can include blue terms from the text. You might also include other important entries you want to have on hand. Having your own glossary can aid your understanding and memorization of the material. It would serve as a great study aid for quizzes and tests, too. We won't call the creation of your own glossary a *requirement*, but it is highly recommended.

Now, it's time to get started. Find a comfortable spot where you can do some amazing thinking. We're excited for all you're going to learn this year, both about logic and about yourself. We hope that you'll enjoy all that you'll learn as much as we teachers and parents will enjoy teaching you!

Chapter 1



Session 1

Read the chapter top to bottom. Best would be to take turns reading it aloud with others. Either way, pause now and then to check your comprehension. "What did those last few paragraphs say?" If unsure, then reread or ask your teacher for help.

Terms & Concepts

Provide concise responses to the following questions.

1. What's logic?

2. What's truth?

3. What's an argument?

4. What's reasoning?

5. What's an inference?

6. What's deduction?

7. What's a principle?

Big Ideas

Respond to the following short-answer questions about chapter content. Answer each question in a few complete sentences.

1. What are three ways that formal logic differs from informal logic?

2. How can formal logic help us discover and communicate truth?

Session 2

Discussion/Demonstration

Answer the following questions, which deepen, extend, or apply chapter content.

- Let's think about Dad's question to Renny after he took a bite of the display sushi: "What was going through your head, Son?" Ren tells the chef that he took a bite of the display food to find out whether he liked sushi. He then tells Dad that the sushi looked real to him, so it seemed reasonable to take a bite of it. We might express Ren's thinking process like this:



Something that looks like sushi is sushi.
 The thing in the display case looks like sushi.
 Therefore, the thing in the display case is sushi.

If this restaurant sets food out on display, then it's okay to sample it.
 This restaurant sets food out on display.
 So, it's okay to sample it.

What do you think about Renny's thinking process? Do his arguments seem reasonable? Did Renny make an error in logic when he chose to take a bite of the display food? (Whatever that answer is, Renny surely made an error in restaurant etiquette!) Write a few sentences to explain whether his thinking process was reasonable.

-
-
2. “Studying logic,” the chapter says, “we learn to see the twin importances of a strong mind and a teachable heart.” What do you think is meant by “strong mind”? What about “teachable heart?” How are these good things? How might studying logic help develop them? Write about 5–7 sentences to explain.

3. What’s “formal” about formal logic relates, in part, to what formal logic isn’t. It “isn’t about using fancy words or eating a salad with a fancy fork.” Formal logic isn’t concerned at all with how we normally use the word “formality.” Rather, it is formal because it pays attention to the form of an argument. Can you think of other situations where the form of some action matters? Does it matter, say, how someone plays a musical instrument, or is what’s played all that matters? Give a few examples of (other) situations where form matters.

Session 3



Discussion/Demonstration

Answer the following questions, which deepen, extend, or apply chapter content.

1. Ever been arguing with someone, and then the argument suddenly stopped making sense? You realized that you weren’t sure where your argument was going. You lost your train of thought and couldn’t even remember where the tracks were! In such a case, it’s likely that the argument lost its

shape, its form—in your mind, at least.

Formal logic helps us understand the shape that reasonable arguments can take. Learning formal logic helps us assess whether an argument is logical. Take the following example.

All spoonbills are animals.
 All waterfowl are animals.
 Therefore, all spoonbills are waterfowl.

Is each sentence true? Yes. Now, look at the form of the argument. Is it logical? The answer is no. Why? What's wrong with it? (Later, you'll learn what's wrong with this argument's form. For now, it's okay to give it your best guess.)

2. Part of formal logic is learning how to express arguments using symbolic language. What advantages might there be to doing so? One comparison that the chapter makes is to mathematical expression.

Imagine I'm hosting a banquet for 63 people and have only nine tables. I'll need to figure out how to divide my guests evenly among the tables. One easy way to express this problem symbolically is $63 \div 9$. How can expressing this seating problem symbolically be helpful? Write a few sentences to explain.



3. Ready to try your hand at writing a formal argument? Let's say you're trying to sort out whether squares are geometric shapes. Following a pattern in the chapter, you write the argument below. In your argument, you substituted "geometric" for "four-sided," which was in the original.

All rectangles are geometric shapes.
 All squares are rectangles.
 Therefore, all squares are geometric shapes.



Each statement in this argument is true. More than that, the argument’s thinking pattern is logical. To see the argument’s thinking pattern, its form, we can write it as below. “R” stands for rectangles, “S” stands for squares, and “G” stands for geometric shapes.

All R are G.
 All S are R.
 Therefore, all S are G.

Use this same good thinking pattern to craft your own argument. Start with an “all” statement (“All __ are __.”) and go from there. Later, you’ll learn the specifics of how to do this. In this first chapter, you’re simply making your best first attempt!

Session 4

Discussion/Demonstration

Answer the following questions, which deepen, extend, or apply chapter content.

1. Consider the following deductive argument. “If an activity is bad for your health, then you shouldn’t do it. Drinking sodas is bad for your health. Therefore, you shouldn’t drink sodas.” The conclusion, that you shouldn’t drink sodas, is an inference. An inference, you’ll recall, is a conclusion reached on the basis of evidence and reasoning.
 What’s the main reason behind this deductive argument’s “guaranteed” conclusion? It’s the



second sentence: “Drinking sodas is bad for your health.” Now, what’s the reason, or supporting evidence, behind *that* claim? How would I know that drinking sodas is bad for my health?

Maybe my dentist told me to lay off the sodas if I want to avoid more cavities! Maybe I watched a health-department video warning about drinking sodas. Maybe I’m a medical doctor who’s seen firsthand how too many sodas can lead to poor health.

“Drinking sodas is bad for your health.” That’s the main reason given for the warning that one shouldn’t drink soda. Here’s the thing: is that reason, that claim, *itself* guaranteed? It seems so, right? Maybe not. Write down two or three reasons for believing that that reason, that claim, might *not* be guaranteed.

In 3–5 sentences, compare my reasons for avoiding soda with your reasons for why drinking it might not be a problem. Then, make a final inference: should drinking soda be avoided or not and why?

2. Deductive reasoning claims that its conclusions are guaranteed. How is this possible? Let’s take an example from the chapter.

If my pet, Macchiato, is a gecko, then Macchiato is a reptile.
 Macchiato is not a reptile.
 So, Macchiato is not a gecko.

Look at the last sentence, the conclusion. Is it merely possible or only probable, or is it guaranteed? If it’s guaranteed, how so? Also, if the conclusion is guaranteed, is it also true? Explain your answers in 3–5 sentences.

3. For all the talk of guarantees, deductive reasoning has a potential “certainty” problem. Whatever claim a deductive argument makes, its supports themselves might not be guaranteed. Consider the following sentence from the chapter. “Deduction’s reasons and supports may come from sense experience.” Sense experience, as life often reveals, can be mistaken. Ever seen how a utensil “bends” in a glass of water?

In the “gecko” argument above, the second sentence says that Macchiato is not a reptile. This assessment is based in part on the speaker’s sense experience, which could be wrong. If the assessment of Macchiato is wrong, then is the argument’s conclusion still guaranteed? What effect would mis-identifying Macchiato have on the truth or falsity of the conclusion? Write 3–5 sentences to explain your thoughts.



Quiz

A. Draw lines to match the following items with their descriptions.

Term or Phrase	Definition or Description
truth	the art or science of reasoning well
logic	an attempt to give reasons or supports for some point of view
principle	kind of reasoning that claims its conclusions are guaranteed, or certain
argument	our mind’s attempt to justify, or prove, the truth of some statement
inference	a statement that conveys a fundamental truth about something
deduction	a conclusion reached on the basis of evidence and reasoning
reasoning	a statement that corresponds to reality, that reflects what is

B. Place a checkmark next to each argument that uses deduction. Don't worry about whether an argument is good or bad or whether its statements are true or false. All we want to know is which use(s) deductive reasoning?

1. I think my foot itches because it was bitten by ants. I stepped in an ant pile yesterday.
2. Mr. Sly must be guilty! He ran from the police, and only guilty people run from the police.
3. All swans that anyone has ever seen are white. All swans must be white, then.
4. No narwhal is a fish, and Nelly is a narwhal. She's certainly no fish, then.
5. I've been hearing weird noises coming from my ceiling. When I went to check the attic, I saw that my dad had set mouse traps. I've probably just been hearing mice scurrying around.

Session 5

Puzzles & Perspectives

Let's see whether we can apply what we've learned to a different scenario. Advertisements on TV and elsewhere often present arguments to try to persuade consumers. Some of these arguments are so bad, one wonders how they got made into commercials! Some ads seem to make sense, though.

Coca Cola, for example, often makes a connection between its products and happiness. One ad shows an ice-cold bottle of Coke. It's dripping with condensation in front of a lush, green field under a bright sun. The slogan next to the bottle says, "Open a Coke, open happiness." This slogan isn't a complete argument, but it has enough information for us to develop one. All we need to do is supply some missing information. We could model the argument as follows.



If you want to be happy, then you should drink a Coke.
You want to be happy.
Therefore, you should drink a Coke.

Does the shape of this argument make logical sense? It does, yes. It follows a good thinking pattern we'll discuss in later chapters. Is the argument's message—its last sentence—right or true? Maybe. The answer depends upon whether the first and second sentences are true.

Now it is your turn. Find an advertisement (a commercial, billboard, radio ad, etc.) and do the following.

1. Describe the ad in a few sentences.

2. Write out the ad's argument in logical form. The argument should have one conclusion, (a kind of "therefore" sentence) and one or more reasons. The Coke argument above gives a good example.

3. Write a few sentences explaining whether the ad's logical *form* seems to make sense. You can tell whether it does by looking at the flow of the sentences. Ask yourself, "If the first two sentences *were* true, would the last sentence *have* to be true, too?" Let's consider a crazy example.

All Zorns are Zanes.
 All Zoobs are Zorns.
 Therefore, all Zoobs are Zanes.

We don't know whether any of these sentences is true. (Let us know if you figure out what Zoobs, Zorns, and Zanes are. We ourselves have no idea!) What we do know is this. If the first and second sentences are (or were) true, then the third sentence would have to be true, as well.

Chapter 2

2

Session 1

Read the chapter top to bottom. Best would be to take turns reading it aloud with others. Either way, pause now and then to check your comprehension. "What did those last few paragraphs say?" If unsure, then reread or ask your teacher for help.

Terms & Concepts

Provide concise responses to the following questions.

1. What's knowledge?

2. What's entailment?

3. What's semantic entailment/consequence?

4. What's logical entailment/consequence?

5. What does it mean to deduce?

6. What is validity? (Or what does it mean for an argument to be valid?)

7. What is soundness? (Or what does it mean for an argument to be sound?)

8. What's a paradox?

Big Ideas

Respond to the following short-answer questions about chapter content. Answer each question in a few complete sentences.

1. Why is entailment necessary for us even simply to go through life?

2. What's the relationship between deduction/deductive reasoning and entailment?

3. What three things are necessary for us to *know* some claim X?

Session 2

Discussion/Demonstration

Answer the following questions, which deepen, extend, or apply chapter content.

1. Entailment is the engine that powers consequence. It's what allows us to draw conclusions or make inferences from other truths we know. Without it, we couldn't have certainty about anything.

Let's take the equation $x + 9 = 12$. To find the value of x , we need to subtract 9 from both sides of the equation. The result is $x = 3$. We could verify our conclusion by plugging 3 in for x in the original equation and seeing if the equation is true. In this case, the new equation would read $3 + 9 = 12$. When we add 3 to 9, we get 12, with the result that $12 = 12$. The equation is true, so our conclusion—that $x = 3$ —was correct.

Now, it's your turn. Solve the following equation and prove that your answer's *certainly* correct: $7z + 6 = 41$. Walk through your solution step by step, as shown above.

2. Entailment says that if some statement X is true, then some statement Y must also be true. Phrased another way, Y's truth is a consequence of X's truth. Let's practice identifying the necessary consequences of various statements. Look at each statement below and then list two or three entailments that follow. We've included an example.





Example:

statement: My dog barked at the man delivering my package.

entailments: I have a dog. My dog can bark. I had a package delivered.

a.

statement: Lisette lost her tennis match yesterday.

entailments:

b.

statement: The cow jumped over the moon. (*Don't overthink this one.*

Put on your hat of imagination.)

entailments:

c.

statement: The earth has a sphere-like shape.

entailments:

d.

statement: Betty Botter bought Bob some butter.

entailments:

3. Remember how we defined entailment? "If some statement X is true, then some statement Y must also be true." Consider this example.

statement X: I got off the plane yesterday in Tulsa.

statement Y: I was in Oklahoma yesterday.

See how the X-to-Y entailment works? If statement X is true, then so is statement Y.

For entailment to work, though, the order of statements matters. Why? What happens if we reverse the statements' order? What if the X statement became the Y statement and vice versa? Let's adjust the previous example.

statement X: I was in Oklahoma yesterday.

statement Y: I got off the plane yesterday in Tulsa.

Can you see that the engine of entailment can break down when the statements' order is reversed? Just because I was in Oklahoma yesterday doesn't mean I got off the plane in Tulsa. I could've gotten off the plane yesterday, but in Oklahoma City.

For each pair of statements below, does the first statement (X) entail the second (Y)? Explain your answers in two or three sentences for each pair.

a.

statement X: I prefer my coffee hot, not iced.

statement Y: I have drunk coffee.

b.

statement X: The element hydrogen is present in the sun.

statement Y: The sun generates its energy from combining hydrogen atoms.

c.

statement X: Shakespeare wrote many works of literature.

statement Y: Shakespeare wrote many plays.

d.

statement X: I have eaten ice.

statement Y: I have consumed water.



- e.
statement X: I can't see very well.
statement Y: I need glasses.

Session 3



Discussion/Demonstration

Answer the following questions, which deepen, extend, or apply chapter content.

1. One kind of entailment is semantic entailment. Statement Y is true because of the meanings of the words in statement X. Take the following example.

statement X: I got off the plane yesterday in Tulsa.
statement Y: I was in Oklahoma yesterday.
explanation of entailment: If I got off the plane yesterday in Tulsa, then I was (certainly) in Oklahoma because Tulsa is a city in Oklahoma.

For the following statements, write one or two sentences each to explain the entailment.

- a.
statement X: Alan got a speeding ticket yesterday.
statement Y: Alan was driving yesterday.
explanation of entailment:

- b.
statement X: Charlotte bought strawberries this morning.
statement Y: Charlotte went to a store, farmers' market, or produce stand this morning.

explanation of entailment:



c.

statement X: Roberto took second place in a pie-eating contest.

statement Y: Roberto ate pie.

explanation of entailment:

d.

statement X: Emma danced at her wedding reception.

statement Y: Emma got married.

explanation of entailment:

2. The other kind of entailment is logical entailment. It says that statement Y is true because of what statement X makes logically necessary. Take the following example.

statement X: Three converging tornadoes destroyed a town on the East Coast last night.

statement Y: Tornadoes exist.

explanation of entailment: Only tornadoes that actually exist could destroy a town, so tornadoes (certainly) exist.

For the following statements, write one or two sentences each to explain the entailment.

a.

statement X: The US state of Georgia is famous for its peaches.

statement Y: Peaches can grow in Georgia.

explanation of entailment:

b.

statement X: Three out of four dentists surveyed say SmileyWhite toothpaste is best.

statement Y: At least four dentists were surveyed.

explanation of entailment:

c.

statement X: Jack and Jill went up the hill to fetch a pail of water.

statement Y: Jill went up the hill.

explanation of entailment:



d.

statement X: Jamal's 14th birthday is today.

statement Y: Jamal was born 14 years ago.

explanation of entailment:

3. Let's say you know something about elves, dwarves, or trolls. Pick one sort of creature and write a few sentences about what you think you know about them. Now, let's say for the sake of argument that elves, dwarves, and trolls don't exist in the real world. Do you still "know" about them what you wrote down? Write a couple of sentences explaining why or why not.

Let's keep assuming that elves, dwarves, and trolls don't exist. Suppose we tell you that we believe dwarves are tall, hairy apes, each with a long, prehensile tail. Are our beliefs about dwarves true or false? If they're false, why? How do you know? Write a few sentences to explain.

Session 4

Discussion/Demonstration

Answer the following questions, which deepen, extend, or apply chapter content.



1. Let's take another quiz, but not on astronomy this time! Mike is taller than Suzy. Jasmine is taller than Suzy, too, but she's shorter than Mike. What are the three kids' relative heights, from shortest to tallest? Easy quiz, right? Here's the real quiz, then. Describe the process of reasoning you used to figure out the kids' heights. Answer in 5-7 sentences.

2. How would you model your reasoning process for each answer? Create a standardized argument that represents your reasoning for which kid is shortest. Create another standardized argument that represents your reasoning for which kid is tallest (To *standardize* an argument is to extract its premises and conclusion from ordinary discourse.) The chapter gives examples of how to do this in the astronomy-quiz section.

3. We don't normally pause in the middle of our thinking to model our reasoning as done in the problem above. What benefit can we get out of doing so, though? How can it help us to show how we came to *know* something as true? Explain your thoughts in 5–7 sentences.

Quiz

Each of the following pairs of statements either do or do not illustrate X-to-Y entailment. For those that do illustrate entailment, it may be semantic or logical. (Some are tricky and could be argued either way.) For each pair below, write SE for semantic entailment, LE for logical entailment, or NE for no entailment. Then, write one or two sentences per pair to explain your answer.



1.
statement X: Cows chew their food.
statement Y: Cows have teeth.

2.
statement X: Several kinds of dinosaurs stood taller than 10 feet.
statement Y: At least two kinds of dinosaurs stood taller than 20 feet.

3.
statement X: The police officer wrote Sinéad a ticket for not stopping at the stop sign.
statement Y: Not stopping at a red light is against the law.

4.
statement X: John ate a steak at a nice restaurant.
statement Y: There is at least one restaurant that offers steaks.

5.

statement X: Mary had a little lamb whose fleece was white as snow.

statement Y: Mary's lamb's fleece was clean.



6.

statement X: Many boats have gone missing in the Bermuda Triangle.

statement Y: At least two boats have gone missing in the Bermuda Triangle.

7.

statement X: The sun's light and heat are necessary for life on earth.

statement Y: The sun's light is necessary for life on earth.

8.

statement X: I ran five miles yesterday.

statement Y: I ran six miles yesterday.



9.

statement X: Last night, I dreamed I was flying.

statement Y: I slept last night.

10.

statement X: Tammy stubbed her toe yesterday.

statement Y: Toes can be stubbed.

Session 5

Puzzles & Perspectives

“This sentence is lying.” Few puzzles can turn the mind to mush faster than the Liar’s Paradox. A paradox, you’ll recall from the chapter, is a statement that seems to say two true but opposite things.

Paradoxes are effective means of messing with people’s minds, but don’t be fooled. They’re no mere party trick. Instead, they give us unexpected, but often frustrating, insight into two important limitations.

The first is the limitation of our own mind. Just because we’ve learned valuable skills in logic doesn’t mean we always know how to use them. Paradoxes can disorient, or even paralyze, our thinking. We understand their words, but what they communicate makes our head spin. These troublesome puzzles remind us to work both on our thinking skills and on our character. They remind us to practice honesty, humility, teachableness, and patience.

The second insight paradoxes provide points to a limitation of logic itself. As powerful a tool as logic is, it can’t sort out every tangle we encounter in thought, language, and life. It’s not the all-purpose hammer before which all the world is a nail. Making sense of our thoughts, feelings, and experiences often requires different tools. As does learning to live in harmony with family, friends, neighbors, and strangers.

True, paradoxes can show us important limitations, both of logic and of our own mind. Still, they provide a helpful playground where we can practice our thinking skills. They give us an opportunity to pit two apparently true claims against each other to see how each fares.

Let’s stack up the two conflicting claims of the Liar’s Paradox as two standardized arguments. The puzzle is so old and complex that different versions of it have appeared over the centuries. Below, we illustrate a version that pits an “honest” sentence against a “dishonest” one. (Note, though, that those “two” sentences are one and the same!) The first argument will suppose that “This sentence is a lie” is telling the truth. The second argument will suppose that “This sentence is a lie” is telling a lie. As in the chapter, we’ll label the arguments’ premises with a P, their conclusions with a C.

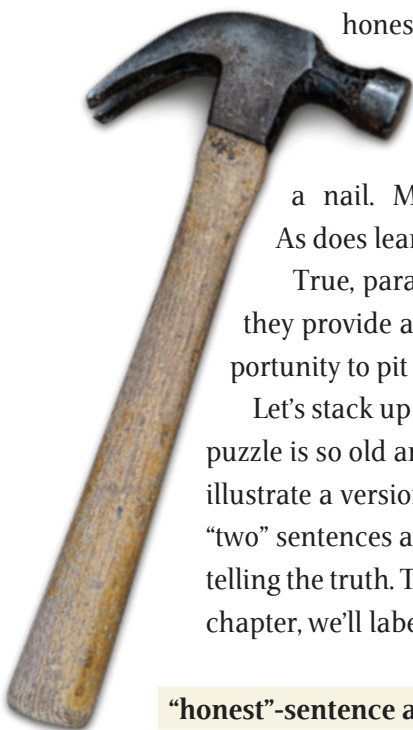
“honest”-sentence argument

- P1** If a sentence tells the truth, then it’s an honest sentence.
P2 “This sentence is a lie” is telling the truth.
C Therefore, “This sentence is a lie” is an honest sentence.

“dishonest”-sentence argument

- P1** If a sentence tells a lie, then it’s a dishonest sentence.
P2 “This sentence is a lie” is telling a lie.
C Therefore, “This sentence is a lie” is a dishonest sentence.

Which is the Liar’s Paradox, then, an honest sentence or a dishonest one? Both arguments make simple but strong cases. “Surely,” the concerned student asks, “only *one* of the arguments is logical and only *one* of the claims is true? Right?” Oh, can a potent paradox set the mind to spinning! We’ll



Chapter 3



Session 1

Read the chapter top to bottom. Best would be to take turns reading it aloud with others. Either way, pause now and then to check your comprehension. "What did those last few paragraphs say?" If unsure, then reread or ask your teacher for help.

Terms & Concepts

Provide concise responses to the following questions.

1. What's an axiom?

2. What's the axiom of identity (ID)?

3. What's the axiom of noncontradiction (NC)?

4. What's the axiom of the excluded middle (EM)?

5. What are the three acts of the mind?

6. What's understanding?

7. What's judgment?

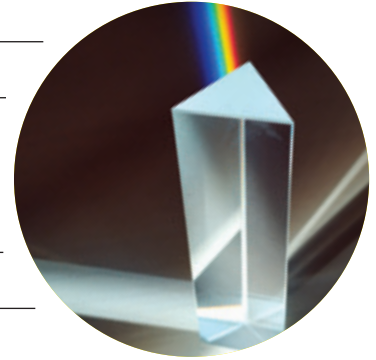
8. What's reason?

Big Ideas

Respond to the following short-answer questions about chapter content. Answer each question in a few complete sentences.

1. Why must we assume that the three axioms of thought are true? Phrased differently, what happens if we don't assume that the three axioms are true?

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2. What roles do the three acts of the mind play in our thinking?
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Session 2

Discussion/Demonstration

Answer the following questions, which deepen, extend, or apply chapter content.

1. In this chapter, we learn about the three axioms of thought. The textbook defines an axiom as “a statement that’s accepted as true, even if it’s neither proven nor provable.” What do you think that last phrase means? Is it possible for a statement not to be provable? Take the axiom of identity, for example. Can we not come up with an argument to prove whether it’s true? If we can’t prove that the ID axiom is true, then why should we believe that it’s true? Write a short paragraph to explain your thoughts.

2. If we don’t assume that ID, NC, and EM are true, then “we’ll be incapable of *any* reasonable thought whatsoever.” Oh? Why not and says who? [Wink.] From what you understand of the three axioms, do you agree with the text? Whatever your opinion, why do you hold it? Share your thoughts in a short paragraph.

3. The ID axiom says that X—whoever or whatever X is—is identical to itself. This may seem obvious, but it’s essential for logical reasoning. Without it, we couldn’t distinguish between different concepts, people, or objects. The axiom of identity helps us note, and insist upon, the unique properties that make a certain X, X. For each set below, identify which property or relationship changes X’s

identity. Also, write a sentence explaining why you chose what you did. Important note: assume that every italicized term below refers to something real, not fictitious. We've done an example for you.

- In the black pouch were *diamonds* that
 - glistened in the sunlight.
 - breathed heavily and coughed hard. (In the real world, diamonds don't breathe or cough.)
 - the thief never found.
 - meant his family's freedom.
- a. The *dog* is
 1. chasing the ball.
 2. brown.
 3. singing in Spanish and Arabic.
 4. howling at the moon.

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- b. The *book* is
 1. resting on the table.
 2. open to page 99.
 3. a paperback.
 4. cooking dinner for the family.

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- c. The *apple* is
 1. actually a chicken.
 2. ripe.
 3. red.
 4. half eaten.

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- d. The *moon* is
 1. appears at night.
 2. is made of cream cheese.
 3. is a sphere.
 4. is full.



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- e. The *car* is
 1. parked.
 2. speeding.
 3. plotting its driver's demise.
 4. in the driveway.
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Session 3

Discussion/Demonstration

Answer the following questions, which deepen, extend, or apply chapter content.

1. The setup to question 3 in session 2 ended with the following sentence. “Important note: assume that every italicized term below refers to something real, not fictitious.” Why was it necessary to include that note? What might have happened if it had been left off? How might you have answered the question differently? Explain your thoughts in a few sentences.

This “important note” raises a curious point, though. We can declare to be true or false even statements about fictitious persons, characters, places, and things. “Achilles was a cowardly weakling.” False. “Aslan is a tame kitty cat.” Rubbish! What if the dog mentioned in session 2 question 3 were the star of a children’s story about opera-singing canines? Wouldn’t knowing that about the dog change our answer to the question? Does the axiom of identity not work, then, if we’re talking about fictitious persons, characters, places, and things? Write a short paragraph to explain your opinion.

2. The EM axiom is important, in part, because it requires every statement to have a truth value of true or false. This will prove super helpful later in the year when we test arguments for validity. We’ll hypothesize about the truth or falsity of premises to see what happens to an argument’s conclusion.

Does EM really apply to every statement, though? Are there no exceptions? What about statements that reflect personal preference? “Vanilla ice cream is the best!” “Bach is better than Mozart any day!” “Bayern München has the best goalie in the league!” Does the EM axiom apply to statements like these? However you answer that question, write a paragraph to clarify and defend your thoughts.

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