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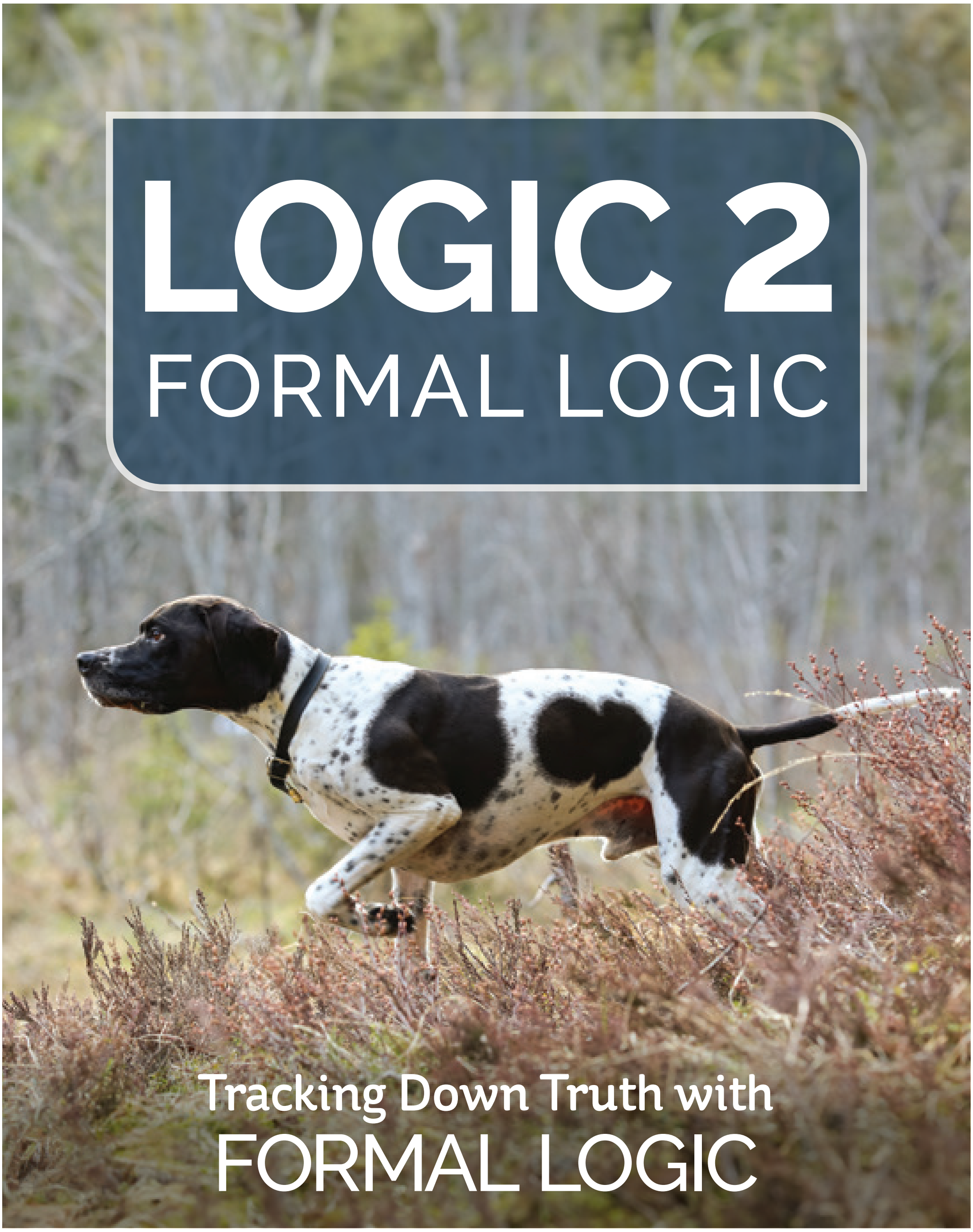
LOGIC 2
FORMAL LOGIC



Tracking Down Truth with
FORMAL LOGIC

MICHAEL G. EATMON
CINDY M. FELSO

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CONTENTS

Part 3: Propositional Logic

Part 4: An Application of Logic

FOREWORD

This, the second of two texts in our logic curriculum, covers formal logic in the most comprehensive way I've seen. I have had the pleasure of being one of the editors and have really enjoyed the side benefit of learning the material. Some are born with a natural inclination toward logical thinking and analysis, and some are not so gifted. Regardless of one's God-given gifts, this text will prepare "all ye who enter" in a remarkable way, empowering them with great ability to reason and avoid shoddy thinking.

Michael Eatmon and Cindy Felso have co-authored a remarkably thorough textbook. You'll see their mastery to the point of making learning difficult concepts fun and even easy. They have been an extraordinary team, and we are very grateful for their work.

Logic, is and will always be, a key discipline in any education. It's crucial in a classical Christian education. Without clear, precise thinking, all the knowledge in the world will do us little good. I hope you enjoy as much benefit from this book and course as we all did in producing it. Students who have imbibed its content will be prepared for life in way that is hard to overestimate. By God's grace it will play a key role in producing a generation of clear thinkers and the kind of leadership that will be part of making things be as God intended.

Join us as we continue to work on

"Restoring Culture to Christ One Young Heart and Mind at a Time."

Marlin Detweiler

President | Veritas Press
May 2024

PREFACE

Welcome to *Logic 2: Formal Logic,* an expansion of the journey we began in *Logic 1: Informal Logic.* That first voyage navigated the deep, often tumultuous seas of everyday reasoning. We focused on cognitive biases and logical fallacies, as well as on how to lessen their impacts. We examined the basics of argumentation and the theory of knowledge, too.

The sequel before you, however, ventures into more *predictable* waters. Here, we'll take deep dives into categorical and propositional logics. We'll also explore logic's foundations, as well as one of its applications, debate.

In today's world, information, misinformation, and disinformation are plentiful. Irrational and unreasonable opinions are even more abundant. What's much less widespread is the ability to reason well. That reality highlights a truth: learning how to use logic isn't a mere academic exercise; it's a vital life skill.

This book is designed to equip young minds and hearts. It focuses on the precise, rule-governed systems of formal logic. The book provides tools for dissecting complex arguments and constructing solid reasoning. It helps prepare students to engage in intellectual discourse both thoughtfully and respectfully.

Logic 2: Formal Logic introduces students to syllogistic and sentential logics. Learning these logics' patterns is foundational to understanding how valid arguments work. Studying categorical and propositional logics develops skills that enrich other studies, too. That's especially true for studies of language and math. What's more, a study of logic enhances one's ability to order the chaos of much modern discourse. Learning logic's patterns helps one cut through the noise of conflicting information.

The book before you also includes short units on the theory of knowledge and the basics of debate. These provide students a broader context for understanding logic and using its skills. Understanding the basis and limits of knowledge is crucial for thoughtful reflection. Learning to defend ideas in a respectful manner is crucial for effective communication.

As with the first book, *Logic 2* makes these complex topics accessible and engaging. We've used clear explanations, practical examples, and interactive exercises. This approach ensures that students' learning is both incremental and comprehensive. We've aimed to win and maintain the attention of our middle-school audience, too.

The authors hope this book will be a guide and a tool. We trust that it will help illuminate a path toward clearer thinking and better arguing. We want students to learn valid patterns of logic and use them. We also want students to appreciate the power of logical thinking in everyday life.

This book would not have appeared when it did were it not for the support of a team. First and foremost, thanks go to Cindy Felso, my coauthor. Her substantial contributions to core content were invaluable. Her careful review of chapters and exercises caught issues before they became problems. Most important, she collaborated with a spirit of grace, gratitude, and generosity. I could not have worked alongside a better co-writer. Thanks go to Carl Petticoffer, too, whose eagle eyes caught things the authors missed. His suggestions were always helpful and kindly delivered. A last thanks goes to Dan Meissner, whose graphics work helped adorn *Logic 2* with an effective beauty.

Michael Eatmon

Orlando, Florida

Part 1:

The Foundations of Logic

LOGIC 2
FORMAL LOGIC

Chapter 1

WHERE THE RUBBER MEETS . . . THE SEA[1]

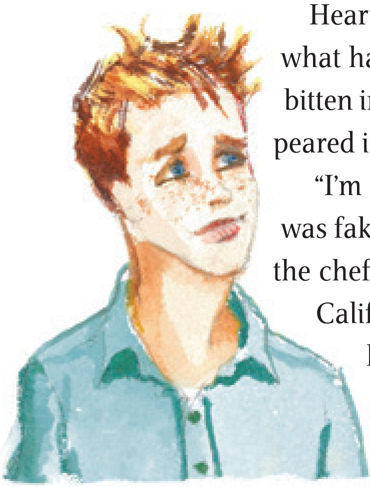
Renny wasn't happy about the restaurant his family was trying out for dinner. "So, it's Jen's birthday. Why did she have to pick sushi?! I suddenly lost my appetite," he complained.

He, Jen, Mom, and Dad walked into the restaurant and waited to be seated. When Renny's stomach started to growl, Jen raised an eyebrow. "Seems you found your appetite."

Renny rolled his eyes but walked over to the display counter to get a closer look.

1 Familiar with the expression "where the rubber meets the road"? It describes "the point at which a theory or idea is put to a practical test" (*New Oxford American Dictionary,* 3rd ed.).

On a whim, he decided to try out some sushi right then and there. He grabbed a California roll and took a big bite. "What is this?!" he roared, and he spat it out.



Hearing the commotion, the hostess walked over. Figuring out what had happened, she laughed. She explained to Renny that he'd bitten into a *plastic* display piece. After seating the family, she disappeared into the kitchen. She soon reappeared with the chef.

"I'm sorry I bit your display," Renny started, "but I had no idea it was fake! I just wanted to see if I liked sushi." Although not amused, the chef accepted the apology and told Renny he'd bring him a real California roll to try. Ren thanked the chef, who returned to the kitchen with a sigh.

"What was going through your head, Son?" Dad asked.

Jen rolled her eyes. "Obviously," she said, "nothing. Who bites into display food?"

"How was I supposed to know it was fake?" Renny protested. "It didn't *look* fake! And why would anyone put fake food on display, anyway? If they want people to see what sushi is like, why not use the real thing?"

"Because it'll go bad just sitting in the case," explained Mom, "and will need to be replaced too often."

"Seems like a trick!" Ren defended himself. "It looks real, but it isn't."

"Oh, it *is* real—real plastic, just not real seafood," Dad said with a smirk.

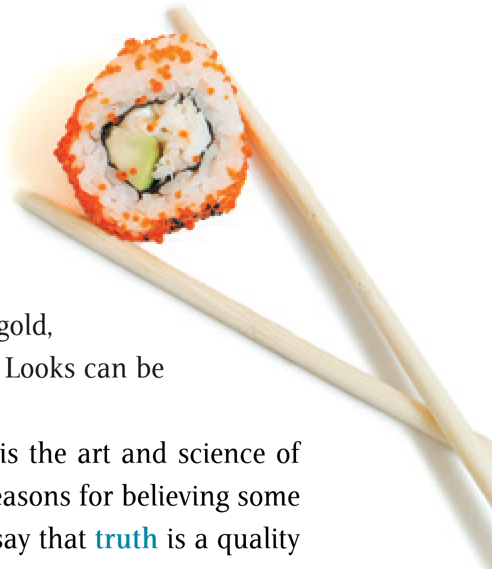
"It *looks* like sushi," Jen agreed, "but it *isn't* sushi in any way that matters at the dinner table."

Surprised to hear an ally in his sister, Ren nodded. "Exactly! Real sushi isn't plastic, and plastic sushi shouldn't look real!"

Dad sat back in his chair, half suppressing a chuckle. "I'm not sure about that, Ren. Something about your reasoning seems, well, fishy."

This time, even Mom rolled her eyes.

Where Truth, Logic and Seafood Meet



Oh, to have been a fly in the van on the way home! "What were you thinking, Ren?!" In other contexts, the question would be rhetorical.[2] In this situation, Mom, Dad, and Jen were genuinely flummoxed. Bite into an unknown substance and hope for the best? Not a winning survival strategy. "You could've broken a tooth, you know . . . or died!" Neither was likely, but Mom made her point. Not all that glitters is gold, nor all that shimmers in the sushi display case, edible. Looks can be deceiving, both in restaurants and in logic.

Logic, you may have learned in an earlier course, is the art and science of reasoning well. It focuses on finding and using good reasons for believing some claim is true. For our purposes in a logic course, we'll say that truth is a quality that some statements possess. For a statement to possess that quality, what it says must reflect reality. What it says must reflect what *is*.[3]

What is **truth?**

For our purposes in a logic course, we'll say that truth is a quality that some statements possess. For a statement to possess that quality, what it says must reflect reality.

We call logic a science because it can help us discover truth about ourselves and the world. We call it an art because with practice, we can improve our use of logic and its tools. An even simpler way to view logic is to see it as a set of thinking patterns. These patterns help us sort orderly, reliable thinking from chaotic, undependable thinking.

2 We pose rhetorical questions for dramatic effect or to make a point, not because we expect an answer.
3 Defining "what is" can be mind-bogglingly complex.

What is **logic?**

It's the art and science of reasoning well. We may also view it as a set of thinking patterns that can help us sort orderly thinking from chaotic thinking.

Studies in logic often divide the subject into two types: formal and informal. Both study thinking patterns. Both aim to sort those that are reasonable and reliable from those that are neither.[4] Each approach to logic aims to expand knowledge, remove errors, and convey truth. Using the tools of either sort of logic will improve both how we think and what we think. Learning to use logic's tools can improve our character, too. Studying logic, we learn to see the twin importances of a strong mind and a teachable heart.

Logic 1, the first in the two-part logic series by Veritas Press, explores informal logic. This type of logic focuses on how our thinking shows up in everyday conversations. Courses in informal logic pay special attention to cognitive biases and informal fallacies. These courses also explore connections between our thinking and reasoning and our emotions. What most distinguishes informal logic, though, is its use of ordinary, natural language. Informal logicians want to discuss our thinking in the language we use to do our thinking.[5]

What's So Formal about Formal Logic?

The book before you, however, is a course in formal logic. This type of logic differs from its informal cousin in at least three important ways. None has to do with how we often think of "formality." Formal logic isn't about using fancy words or eating a salad with a fancy fork.

Before we summarize what distinguishes the two types of

4 If a thinking pattern is reasonable, then it's "based on good sense" (*New Oxford American Dictionary,* 3rd ed.). When a thinking pattern is reasonable, it's more likely to lead us to truth.

5 A logician is someone who studies logic or is an expert in the subject.

logic, we need to say a few words about arguments. "Huh? Why are we talking about bickering and brawling in a logic course?" Given how many people use the term argument, that's a good question. When logicians use the word, they don't mean a disagreement or a fight. Instead, they mean an attempt to give reasons or supports for some claim, some point of view.

What is an **argument?**

It's an attempt to give reasons or supports for some claim, some point of view.

Back to our question: how does formal logic differ from its informal cousin?

First, formal logic pays most attention to an argument's form, or shape. To a formal logician, what an argument is trying to prove is only of secondary importance. Arguments with particular forms, or shapes, are seen as possessing good thinking patterns. Other arguments, those with defective forms, are seen as faulty. If an argument is put together in a particular way, says the formal logician, it'll be logical. For the curious student who's now wondering, no, "logical" isn't a synonym for true. We'll have more to say on that topic in later chapters. If an argument has a faulty form or shape, it'll be flawed, regardless of its content.

LOGICAL ≠ TRUE

Let's look at a couple of examples. Compare this argument

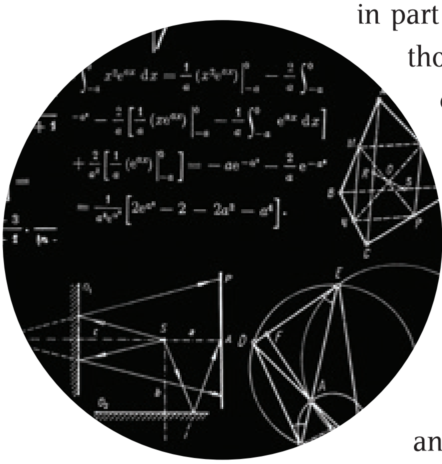
All rectangles are four-sided shapes.
All squares are rectangles.
Therefore, all squares are four-sided shapes.

to this one

All squares are four-sided shapes.
All rectangles are four-sided shapes.
Therefore, all squares are rectangles.

What do you think: do both arguments follow a good thinking pattern? Would it help to know that each statement in each argument is true? "If every statement's true, then . . . yes, both arguments show good thinking." How bold the student who ventures such a guess, especially so early in the course!

Fortune may favor the brave; logic, alas, does not. The first argument's pattern is reasonable and reliable. It's such a reliable thinking pattern that some logicians give it a special name, "Barbara." We'll say more about Barbara and her friends in part 2. What about the second argument's thinking pattern, though—reasonable, reliable? The short answer is no, but its explanation will need to wait, as well.



The second way formal logic differs from its cousin is in the language it uses to talk about arguments. Informal logic uses ordinary language to create and assess arguments. Formal logic, however, often uses languages—sets of symbols and rules—that look more like math. Some students just winced, but they needn't worry. They'll soon see that formal logic's symbols and rules aren't so scary after all.

Ready to take a peek at some of the symbolic language you will encounter this year? Reread the square-and-rectangle arguments above. Now, let's translate them into one of the common languages of formal logic.

Here's the first square-and-rectangle argument, the one with the good thinking pattern.

All R are F.
All S are R.
Therefore, all S are F.

And here's the second, the one that won't win any awards for logical reasoning.

All S are F.
All R are F.
Therefore, all S are R.

Surprised by how much formal "logic-ese" you're already able to read? Once you find out what those Rs, Fs, and Ses stand for, you'll be amazed at how fluent you can be in the language of formal logic.

The Little Engine that Could

The third important way in which the two types of logic differ relates to how we put arguments together. Each sort of logic tends to focus on different kinds of reasoning. Reasoning is what our mind is doing when it argues, when it tries to justify, or prove, the truth of some statement.

What is **reasoning?**

It's what our mind is doing when it tries to justify, or prove, the truth of some statement.

"People shouldn't take a bite out of restaurants' display food. It may be plastic, and chewing plastic isn't a pleasant experience." The first sentence states a viewpoint. The second gives reasons for believing the viewpoint is sensible or true. The mental engine that powered the two-sentence argument was restaurant-savvy reasoning.

Logicians have long recognized different kinds of reasoning. They often disagree about the details, though. How many different kinds of reasoning are there? How can we define and differentiate them? What sorts of inferences does each allow us to make? (An inference is a conclusion reached on the basis of evidence and reasoning.) Experts have argued over the answers for centuries. We won't be settling the matter in a middle-school logic course.

What is an **inference?**

It's a conclusion reached on the basis of evidence and reasoning.

The kinds of reasoning that tend to show up in informal logic are induction and abduction. *Logic 1* has much to say about those topics, so we won't repeat the discussion here. The kind of reasoning that shows up most often in formal logic, however, is deduction. Deductive reasoning is unique in that its arguments' conclusions claim to be *guaranteed*. Read that last word again: not "possible," not "probable," but "guaranteed."

What is **deduction?**

It's a kind of reasoning that claims its conclusions are guaranteed, or certain.

Induction, abduction, and deduction can look alike in at least one important way. The reasons and supports they use in their arguments may come from sense experience. Sometimes, though, deductive arguments rely for support on one or more principles. A principle states a fundamental truth about something. Principles arise from a general understanding of an idea, not from sense experience. For example, "a triangle may have no more than one right angle." Mathematicians agree that no triangle may have more than one 90° angle. No one, though, has measured every angle in every triangle just to be sure.

What is a **principle?**

It's a statement that conveys a fundamental truth about something. Principles arise from a general understanding of an idea, not from sense experience.

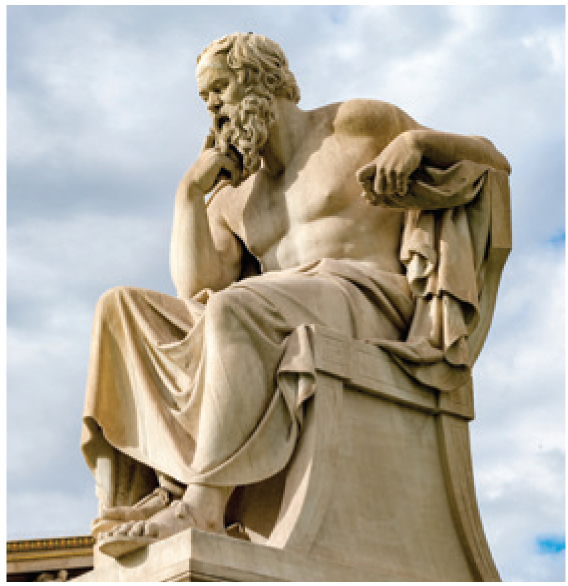
Want an example of deductive reasoning so you can wrap your head around how its guarantee works? Grab your mental chopsticks and sample this morsel.

If my pet, Macchiato, is a gecko, then Macchiato is a reptile.
Macchiato is not a reptile.
So, Macchiato is not a gecko.

If Macchiato isn't a reptile, the argument claims, then Macchiato isn't a gecko—guaranteed. "Wait," a perceptive student demurs. "Are you saying that every deductive argument's conclusion is guaranteed? Or that every claim that every deductive argument makes is true?" No and no. In fact, whether a deductive argument's claim is either guaranteed or true can be arguable. Those two topics we'll take up in the next chapter.

Chapter 2

ONE THING LEADS TO ANOTHER



Philosophers can be a contentious crowd, but no wonder! They contemplate some of life's most important questions. What's really real? What's the difference between right and wrong? How can we know things?

It's that last question that piques the curiosity of many logicians. What *is* knowledge, and how can we know when we have it? To philosophers and logicians, it isn't a feeling, a hunch, or a wish. Instead, **knowledge** is justified true belief, and each word in that definition is vital.[1]

What does it mean for us to *know* something—some statement S?

First, it means that *we believe S is true*. Second, it means that *S is in fact true*. Third, it means that *we have good reason to believe S is true*.

Experts in logic and philosophy bicker and argue about a lot. What they don't disagree about is the role that *entailment* plays in deduction. **Entailment** is what allows us to *know* that if statement X is true, then statement Y must be true, as well.

1 *Logic I: Informal Logic* (Veritas Press) has much more to say about the topic of knowledge.

What is **entailment?**

It's what allows us to know that if statement X is true, then statement Y must be true, as well.

Without entailment, deductive reasoning would be impossible. All our arguments would need to rely on induction or abduction. This would be fine if all we needed were to forecast the weather or explain broken windows. Our arguments could squeak by with merely probable conclusions. What if we wanted to create an argument whose conclusion was guaranteed, though? Without entailment, it couldn't happen.

Worse, any area of human life that requires the use of guaranteed reasoning would cease. Common uses of natural language wouldn't work as expected, and most uses of math and science would stop. Even simple algebraic problems would stump us forever. What does x equal in the equation $9 + x = 12$? We could guess, but we couldn't know for sure. We might stumble onto $x = 3$, but without entailment, we'd have no way to check our answer. Without entailment, our reasoning processes couldn't be *certain* about *anything*.

"Entailment sounds super important!" gathers the attentive student. "I can't imagine what life would be like without it . . . but mostly because I'm not sure what it *is*." We appreciate the curious student's enthusiasm and engagement. We appreciate her intellectual honesty, too.[2]

Chain Reactions

Wrapping the mind around what entailment is can be a challenge. We could ask a logician for a definition, but trying to understand it would be like jogging through glue. Let's take an easier, more metaphoric approach. We can think of

2 Intellectual honesty is a truthfulness that applies to how we use our mind. If we're intellectually honest, we won't pretend to know or understand X when in fact we don't.

entailment as one of two engines that power *consequence*. Those two engines are *semantic* consequence and *logical* consequence.[3]

When we say that Y's truth is a semantic consequence of X's truth, what do we mean? We mean that statement Y is true because of the meanings of the words in statement X. Consider an example.

Statement X: Jack won the game.
Statement Y: Jack played the game.
explanation of X-to-Y entailment: If Jack won the game, then he (certainly) played the game.

Happen to notice the order of the entailment above? X entailed Y. What do you imagine would happen if we reversed the order? Would the Y statement entail the X statement? Is entailment like a reversible belt, workable both ways?

We can find out with a simple test. Let's swap the order of the X and Y statements. What do we get? "If Jack played the game (Y), then he won the game (X)." Does that swapped-order statement *have* to be true? No, the swapped-order statement *may* be true, but it wouldn't *have* to be true. Jack's mere playing of the game is no guarantee of his winning it. One can play a game without winning it, but one can't win a game without playing it. Semantic entailment is a powerful engine, but it is guaranteed to work in only one direction.

3 *Semantic* is a fancy word that means "having to do with the meanings of words."

What is **semantic consequence** or **semantic entailment?**

It's the engine that powers the following relationship: Statement Y is true because of the meanings of the words in statement X.

The other type of entailment engine is logical consequence. When we say that Y's truth is a logical consequence of X's truth, what do we mean? We mean that statement Y is true because of what statement X makes logically necessary. That may sound confusing, but an example should make the relationship clear.

Statement X: Sarah and Sally sold seashells by the seashore.
Statement Y: Sarah sold seashells by the seashore.
explanation of X-to-Y entailment: If both Sarah and Sally sold seashells by the seashore, then Sarah sold seashells by the seashore.

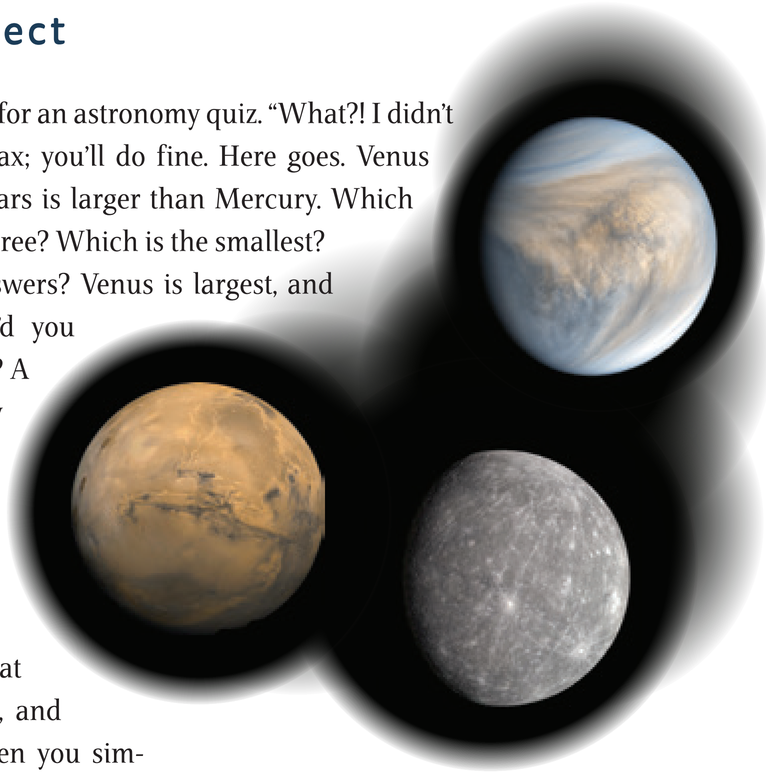
As with semantic entailment, the order of statements here matters. We can verify that by swapping the order of the X and Y statements. "If Sarah sold seashells by the seashore (Y), then Sarah and Sally sold seashells by the seashore (X)." Does this swapped-order statement have to be true? It doesn't, no. Sarah could've sold the seashells by the seashore all by herself. In this pair of statements, X entails Y, but Y doesn't entail X.

What is **logical consequence** or **logical entailment?**

It's the engine that powers the following relationship: Statement Y is true because of what statement X makes logically necessary.

A perceptive student is now cocking his head to one side, his mind glimpsing a keen insight. "Entailment lets us say that if some statement X is true, then some statement Y must also be true. I get that, but doesn't entailment work only if we know, or at least assume, that statement X is true in the first place?" The light bulb over that kid's head is shining bright.

The Domino Effect



Grab a pen and paper. Time for an astronomy quiz. "What?! I didn't get a chance to study!" Relax; you'll do fine. Here goes. Venus is larger than Mars, and Mars is larger than Mercury. Which planet is the largest of the three? Which is the smallest?

Ready to check your answers? Venus is largest, and Mercury is smallest. How'd you do? A perfect score, you say? A career in space science may be in your future!

Now, let's explore how you did so well on the quiz. Did you happen to know the three planets' approximate diameters: Venus at 3,800 miles, Mars at 2,100, and Mercury at 1,500? If so, then you simply compared numbers. The more probable explanation for your acing of the quiz wasn't prior knowledge, though.

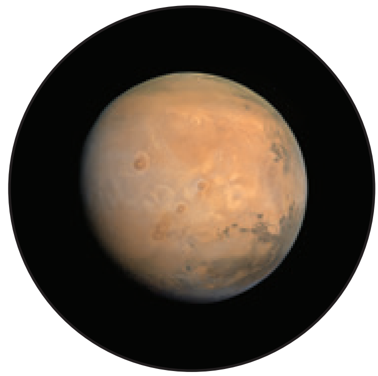
It was a two-step reasoning process that required no knowledge of astronomy whatsoever. First, you presumed to be true what the quiz's setup stated about the planets' relative sizes. Second, you fired up the engine of entailment.

Here's how you likely used entailment for today's astronomy quiz; you reasoned as follows. "Let's presume, as the quiz says, that Venus is larger than Mars and Mars is larger than Mercury. If those two relationships are true, then Venus must be larger than Mercury, as well. And if that's true, then Venus is the largest of the three." Turning the reasoning around, you also inferred the smallest of the three, Mercury. Here are two ways to represent the sort of reasoning you may have used.[4]

4 The four arguments here show their premises (P) and conclusions (C) in standard order. If you have taken a previous logic course, this arrangement will look familiar. A *premise,* you'll likely recall, is a statement that supports, or helps to support, an argument's conclusion. A conclusion is the statement said to be supported or proven by an argument's premise(s).

Largest-planet argument (I)

Premise 1 Venus is larger than Mars.
Premise 2 Mars is larger than Mercury.
Conclusion 1 Venus is larger than Mercury.[5]
Conclusion 2 So, Venus is the largest of the three.



Smallest-planet argument (I)

P1 Mercury is smaller than Mars.
P2 Mars is smaller than Venus.
C1 Mercury is smaller than Venus.
C2 So, Mercury is the smallest of the three.

Largest-planet argument (II)

P1 If Venus is larger than Mars and Mars is larger than Mercury, then Venus is the largest of the three.
P2 Venus is larger than Mars, and Mars is larger than Mercury.
C So, Venus is the largest of the three.

Smallest-planet argument (II)

P1 If Mercury is smaller than Mars and Mars is smaller than Venus, then Mercury is the smallest of the three.
P2 Mercury is smaller than Mars, and Mars is smaller than Venus.
C So, Mercury is the smallest of the three.

Notice that you needed no factual knowledge of the planets beyond what was provided. You didn't need to know their diameters or distances from the sun. All you needed was to presume the truth of two implied arguments' premises. One argument lets you deduce which planet is largest; the other argument, which is

5 You can think of Conclusion 1 (C1) as an intermediate conclusion and Conclusion 2 (C2) as a final conclusion.

smallest. (To deduce is to see, or to infer, the necessary consequence of one or more statements.) Deductive entailment works much like the automatic action of falling dominoes. Line up the dominoes close together and knock one over. Others will fall as a result.

What does it mean to **deduce?**

It's to see, or to infer, the necessary consequence of one or more statements. That consequence may be semantically entailed or logically entailed.

A Money-Back Guarantee of Truth?

A few astute students are still pondering our domino analogy. "I follow how deductive entailment works if we assume an argument's premises are true. What if one or more premises aren't actually true, though? Does deductive reasoning still work?" What a perceptive question! It's also predictive. It points toward where our conversation is going next. When we're discussing deductive arguments, we need to distinguish *validity* from *soundness*. Logicians use those two terms often, so it's essential to grasp their meanings.

What do we mean when we say that an argument is valid? We mean that its conclusion follows *necessarily* from its premises. Phrased traditionally, it's *impossible* for the premises to be true but the conclusion to be false.

What does it mean for an argument to be **valid?**

It means that its conclusion follows *necessarily* from its premises. It's *impossible*, that is, for the premises to be true but the conclusion to be false.

When we examine an argument for validity, we focus on its thinking pattern. If the premises *were* true, we ask ourselves, would the conclusion *have* to follow? Assessing for validity ignores the *actual* truth or falsity of the argument's content. Let's consider an example.

P1 All pangolins can fly.
P2 Percy is a pangolin.
C So, Percy can fly.

However fanciful the argument, it *is* valid. If the premises *were* true, the conclusion would *have* to be true, as well.

Now, let's voice what many students are doubtless thinking. "But pangolins can't fly!" Right, and that objection brings us to the second term we mentioned—soundness. What do logicians mean when they say that an argument is sound? They mean both that the argument is valid and that its premises are true. Sound arguments lead to true conclusions, and we can *know* that those conclusions are true.

What does it mean for an argument to be **sound?**

It means both that the argument is valid and that its premises are true.

This distinction between validity and soundness highlights a limitation of deductive logic. Deductive reasoning *alone* can't determine an argument's soundness, only its validity. To assess soundness, we need knowledge about the actual truth or falsity of the premises.

sound argument = valid argument + true premises

Logic is a powerful tool, but it isn't omnipotent. Whereas deductive entailment can ensure validity, it can't guarantee soundness. Establishing an argument's truth claims requires more than logic. It requires observation, analysis, and reflection. It can also call for healthy doses of curiosity, humility, and teachableness. Master logicians appreciate the power of logic to help lead us to truth. They also understand that logic alone is rarely enough to get us there.

A Tale of Two Truths

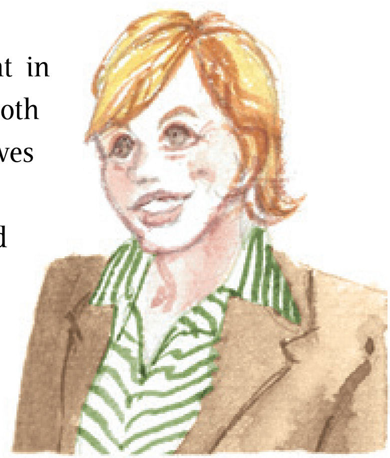
"The evidence is incontrovertible: that cow was abducted!"

José, Renny, and extraterrestrial beings had a history. It'd been months, though, since José breathed a word about alien activity. Renny figured José's UFO phase had passed, but today's comment proved otherwise.

"You're probably wrong," Ren replied, "but nice use of 'incontrovertible.'" The boys disagreed about UFOs, but both appreciated a well placed $5 vocabulary word.

Overhearing them, Mrs. Sagewright couldn't pass up an opportunity to bend a brain or two. "José, Renny, may I ask you a question?"

The boys zipped their lips and nodded.



"What if you're both right?" Seeing bewilderment in the boys' eyes, Mrs. Sagewright continued. "You've both thought a lot about the subject, and each of you believes he's right. Maybe, each of you *is* right."

A few seconds later, her suggestion sank in, and José opened his mouth. "I'm confused. Are you saying that maybe I'm right that the cow was abducted *and* Renny's right that it wasn't?"

"That's what I'm wondering," she replied.

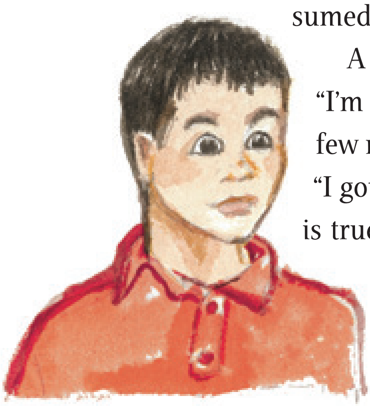
Renny cracked a smile, imagining he'd discovered the game. "This is one of those paradoxes, isn't it?"

What is a **paradox?**

It's a statement that seems to say two true but opposite things.

“No,” she assured him, “but it is a puzzler.” She waited a bit to see whether a light bulb appeared over either boy. No such glow of understanding showed up. “How about an easier question? Let’s say José claims it’s pouring outside, and Renny claims it’s not raining at all. Couldn’t you both be right?”

Renny’s brain was spinning its wheels, a feeling both uncommon and uncomfortable. He mumbled a few words to himself and shook his head. He still assumed this was a game, but he had no idea how to play it.



A different feeling came over José, a feeling of determination. “I’m going to figure this out,” he murmured, his brow furrowed. A few more seconds of silence passed, both boys still in the hot seat. “I got it!” José blurted. “Either what I say about that abducted cow is true, or what Renny says about it is. We can’t both be right; it’s impossible. It’s not . . . allowed!”

“Oh?” Mrs. Sagewright intoned with a playful grin. “Not allowed?! Says who?”

Chapter 3

A THREEFOLD BEAM OF LIGHT

Ready for a short art quiz on the color wheel? What do you get when you mix red and yellow paint? “That’s easy. Orange.” Clearly, you studied, so here comes a challenge question or two. What do you get when you blend equal parts red, yellow, and blue paints? “Nice try, but brown.” One last question, then: what do you get when you blend equal parts red, green, and blue light? “That seems trickier, but I’ll say *dark* brown.” Sorry, but no.

Even if you missed that last question, you still passed the quiz with flying colors. Happen to catch what made the last question tricky? It asked about light, not paint. Mixing colors of light can produce unexpected results. Nowhere is this more obvious than when blending light’s primary colors: red, green, and blue. When combined, they produce *white* light.

Something similar happens when we blend three different “colored beams” in logic. They produce a bright, white light that allows us to see and assess an argument’s reasonableness. Some philosophers refer to these three beams as the *laws* of thought, but we’ll refer to them as *axioms*. By axiom, we mean a

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